RAN-2203000205023006

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T.Y.B.Sc. (Sem. V) Examination October - 2025

Mathematics : MTH - 506 (Old)

Number Theory - I

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the above details on your answer book

(2) All questions are compulsory.

(3) Figures to the right indicate marks of the question.

(4) Follow usual notations.

Seat No.:

Student's Signature

Q-1. **Answer Five any of the following:** **(10)**

(1) For any integer, show that gcd $(2a + 1, 9a + 4) = 1$.

(2) If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$ then prove that $a + c \equiv b + d \pmod{n}$.

(3) If 1 is added to the product of twin prime than a perfect square is always obtain

(4) Determine all the solutions in the integers of the Diophantine equation $5x + 10y = 12$.

(5) Prove that $89 \mid 2^{44} - 1$.

(6) If a and b are an odd integers then prove that $16 / a^4 + b^4 - 2$.

(7) Find the prime factorization of 899.

(8) If p is a prime and $p \mid ab$ then prove that $p \mid a$ or $p \mid b$.

Q-2. **Answer any two questions:** **(10)**

(1) Find integers x and y such that gcd $(1206, 6540) = 1206x + 6540y$.

(2) For positive integers a and b prove that gcd (a, b) lcm $(a,b) = a.b$.

(3) If a is any odd integer, then prove that $32 \mid (a^2 + 3)(a^2 + 7)$.

Q-3: **Answer any two questions:** **(10)**

(1) Determine all the solutions in the integers of the Diophantine equation $14x + 35y = 91$.

(2) (i) If $p \geq 5$ is a prime then show that $p^2 + 2$ is composite.
(ii) Prove that the only prime of the form $n^3 - 1$ is 7.

(3) Prove that if p is prime then $\sqrt{p}$ is an irrational number.

Q-4: **Answer any two questions:** **(10)**

(1) If p_n is the n^{th} prime number then prove that $P_n \leq 2^{2^{n-1}}$.

(2) Prove that there is an infinite number of primes of the form $4n + 3$.

(3) Prove that the sequence $(n + 1)! - 2, (n + 1)! - 3, \ldots, (n + 1)! - (n + 1)$ produce a consecutive composite numbers for $n \geq 1$.

Q-5: **Answer any two questions:** **(10)**

(1) Working modulo 9 or 11, find the missing digits in the following calculation: $51840 \times 273581 = 1418243\ x040$.

(2) Prove that the integer $53^{103} + 103^{53}$ is divisible by 39.

(3) For any integer a and b show that $a \equiv b \pmod{n}$ if and only if a and b leaves the same reminder when divided by n.